Truthmakers

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Philosophical Logic 2024
19 November 2024

Readings

Required:

- Lecture notes: chapter 5

Optional:

- Rodriguez-Pereyra, G. (2006). Truthmakers. *Philosophy Compass*, 1(2), 186-200.

Plan

1. Truthmaker
2. Relevance
3. Truthmaker semantics

Outline

What is a truthmaker?

We have certain sentences

(1) *The grass is green.*

(2) *I am here.*

(3) *Dinosaurs existed.*

(4) *Unicorns do not exist.*

We say that they are true.

But what makes them true?

'I am here' is true because I am here in the real world; or I am here in the real world because 'I am here' is true?

The former is the enterprise of truthmaker theories.

Some important questions

Are truths all the same? Do all of them have truthmakers?

How can a truthmaker *make* something true (dependence, grounding, supervenience, . . .)

What kind of things are truthmakers (states of affairs, tropes, counterparts, . . .)

Which truths have truthmakers?

Maximalism: The thesis that all truths have truthmakers (Armstrong 2004).

Non-Maximalism: The idea that some truths lack truthmakers. There are truthmaker gaps. (Cameron 2008)

Challenges to Maximalism

It has an intuitive appeal.

But

- Negative truths:
  'There are no unicorns'
  'The moon is red'
  $\rightarrow$ Russell (1918): negative facts
- M: 'This sentence has no truthmaker'.

Milne (2005): Suppose that M has a truthmaker. Then it is true. So what it says is the case is the case. Hence M has no truthmaker. On the supposition that M has a truthmaker, it has no truthmaker. By reductio ad absurdum, M has no truthmaker. But this is just what M says. Hence M is a truth without a truthmaker.

The truthmaking relation

What makes a truthmaker to be the truthmaker of a particular sentence?

Truthmaker as *entailment*:

If x truthmaker for p and p entails q then x is also a truthmaker for q.

What is the problem with this proposal?

A problem

1. $p \vee \neg p$ is a necessary truth
2. Any x that exists can be a truth-maker for it.
3. Something makes a disjunction true by making one of the disjuncts true.
4. Suppose p. Then $\neg p$ cannot be.
5. Thus x is a truthmaker of p [??]

Truthmaking in terms of aboutness/relevance.

[Restall 1996]

Outline

Lewis Argument

1. $p \wedge \neg p$
2. p ($\wedge$ elimination)
3. $\neg p$ ($\wedge$ elimination)
4. $p \vee q$ ($\vee$ introduction)
5. q (disjunctive syllogism)

What to give up?

- $\wedge$ elimination
- $\vee$ introduction
- disjunctive syllogism
- transitivity of entailment

Tautological Entailment

We will describe a procedure to define a notion of **tautological entailment** that captures 'relevant' ones.

Literals: p, $\neg q$, ...

Primitive con/disjunction (con/disjunction of literals): $p \wedge q$, $p \vee r$ [not $(p \wedge q) \vee r$].

Explicit tautological: $\phi \models_{ET} \psi$ iff

1. ϕ is a primitive conjunction and ψ is a primitive disjunction
2. Some literal of ϕ is identical to some literal of ψ

$p \wedge \neg p \wedge q \models_{ET} p \vee r$

$p \wedge q \not\models_{ET} r$

General Case

Consider two formulas ϕ and ψ. We want to check whether ϕ is a tautological entailment of ψ: $\phi \models_T \psi$

1. Put the premise ϕ in disjunctive normal form[1]

2. Put the conclusion ψ in conjunctive normal form

3. Hence, we can rewrite $\phi \models_T \psi$ as

$$\phi_1 \vee \ldots \vee \phi_n \models_T \psi_1 \wedge \ldots \wedge \psi_m$$

4. $\phi \models_T \psi$ holds iff for every ϕ_i and ψ_j, we have $\phi_i \models_{ET} \psi_j$

[1] First use De Morgan's laws and double negation. Then use distribution and commutation. And then use association to group formulas on the left. Normals forms are not unique, but one (un)successful normal form is sufficient.

Converting into Normal Form

Commutation

$$\phi \wedge \psi \iff \psi \wedge \phi$$
$$\phi \vee \psi \iff \psi \vee \phi$$

Association

$$(\phi \wedge \psi) \wedge \chi \iff \phi \wedge (\psi \wedge \chi)$$
$$(\phi \vee \psi) \vee \chi \iff \phi \vee (\psi \vee \chi)$$

Distribution

$$\phi \wedge (\psi \vee \chi) \iff (\phi \wedge \psi) \vee (\phi \wedge \chi)$$
$$\phi \vee (\psi \wedge \chi) \iff (\phi \vee \psi) \wedge (\phi \vee \chi)$$

Double Negation $\neg(\neg\phi) \iff \phi$

De Morgan's Laws

$$\neg(\phi \wedge \psi) \iff \neg\phi \vee \neg\psi$$
$$\neg(\phi \vee \psi) \iff \neg\phi \wedge \neg\psi$$

An example

We check whether $\neg(\neg p \wedge \neg q) \models_T p \vee \neg(r \wedge \neg r)$

Generate the normal form:

$p \vee q \models_T p \vee \neg r \vee r$

We have

$p \models_{ET} p \vee \neg r \vee r$

$q \not\models_{ET} p \vee \neg r \vee r$

Hence,

$\neg(\neg p \wedge \neg q) \not\models_T p \vee \neg(r \wedge \neg r)$

Some further examples

Tautological entailment captures relevance.

$p \wedge \neg p \not\models_T q$

$p \not\models_T q \vee \neg q$

$p \wedge \neg p \not\models_T q \wedge \neg q$

Disjunctive syllogism does not hold

$p \wedge (\neg p \vee q) \not\models_T q$

$->$ Lewis argument is blocked.

Axiomatic First Degree Entailment

The logic of tautological entailment can be captured by an axiomatic system (related to a system developed by Ackermann in the '30).

Transitivity: From $\phi \models_T \psi$ and $\psi \models_T \chi$, infer $\phi \models_T \chi$.

Conjunction: Axioms: $\phi \wedge \psi \models_T \phi, \quad \phi \wedge \psi \models_T \psi$.
Rule: From $\phi \models_T \psi$ and $\phi \models_T \chi$, infer $\phi \models_T \psi \wedge \chi$.

Disjunction: Axioms: $\phi \models_T \phi \vee \psi, \quad \psi \models_T \phi \vee \psi$.
Rule: From $\phi \models_T \chi$ and $\psi \models_T \chi$, infer $\phi \vee \psi \models_T \chi$.

Distribution: $\phi \wedge (\psi \vee \chi) \models_T (\phi \wedge \psi) \vee \chi$.

Negation: Axioms: $\phi \models_T \neg\neg\phi, \quad \neg\neg\phi \models_T \phi$.
Rule: From $\phi \models_T \psi$, infer $\neg\psi \models_T \neg\phi$.

Exercise

Check that $\phi \wedge \neg\phi \not\models_T \psi \vee \neg\psi$

Adding it as axiom results in the RM (R-mingle) logic.

Four-valued logic FDE

The same logic can be captured using four-valued truth tables.

We have already seen such tables. It is FDE! Here, we are employing $\{1\}, \{0\}, \varnothing, \{1,0\}$ for $1, 0, n, b$.

ϕ	$\neg\phi$
$\{1\}$	$\{0\}$
$\{0\}$	$\{1\}$
$\emptyset$	$\emptyset$
$\{1,0\}$	$\{1,0\}$

$\wedge$	$\{1\}$	$\{0\}$	$\emptyset$	$\{1,0\}$
$\{1\}$	$\{1\}$	$\{0\}$	$\emptyset$	$\{1,0\}$
$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$
$\emptyset$	$\emptyset$	$\{0\}$	$\emptyset$	$\{0\}$
$\{1,0\}$	$\{1,0\}$	$\{0\}$	$\{0\}$	$\{1,0\}$

$\phi \models_T \psi$ iff for all valuations v, we have

1. if $1 \in v(\phi)$, then $1 \in v(\psi)$, AND
2. if $0 \in v(\psi)$, then $0 \in v(\phi)$

Outline

1. Truthmaker

2. Relevance

3. Truthmaker semantics

Hyperintesionality

FDE provides an *extensional* semantics (we just look at *the* value assigned to a sentence in a model).

Intensional semantics uses possible worlds, but this is still not *fine-grained* enough.

(1) Venus is Venus.
(2) Venus is the morning star.

(1) and (2) are necessarily equivalent (true in the same possible worlds)

We need a **hyperintensional** semantics.

[Berto and Nolan 2023]

The theoretical landscape

Different ‘truthmaker’ theories have been proposed over the years.

1. Russell atomism and Wittgenstein Tractatus
2. Van Fraassen’s truthmakers
3. Situation semantics
4. Awareness logics
5. Data semantics
6. Recent truthmaker semantics
7. Inquisitive semantics
8. …

A note

We will see a variety of ways to formalize truthmaker semantics.

The point is not to confuse you. But you should get used to the fact that the literature (not just on truthmaker semantics) exhibits a variety of different notations and formalizations, and you should be able to navigate through them and recognize that they amount to the same formalism.

Ontology

State of Affairs $\approx$ literals of formal language
$\mathbf{p}, \overline{\mathbf{q}}, \langle \mathbf{P}, \mathbf{d_1} \cdots \mathbf{d_n} \rangle$

We assume that for every $\mathbf{p}$ in SOA, we also have a complement $\overline{\mathbf{p}}$, for which it holds that $\overline{\overline{\mathbf{p}}}$ is equivalent to $\mathbf{p}$.

Facts $=$ Sets of SOAs $\{\mathbf{p}, \overline{\mathbf{q}}\}$

p is not only made true by $\{\mathbf{p}\}$, but also (inexactly) by $\{\mathbf{p}, \mathbf{q}\}$

Propositions $=$ Sets of Facts $\{\{\mathbf{p}, \mathbf{q}\}, \{\mathbf{p}\}\}$ [the fact which makes p true and q false]

Van Fraassen's truthmaker semantics

Exact Truthmaker semantics

$$
\begin{array}{ll}
T(p) = \{\{\mathbf{p}\}\} & F(p) = \{\{\overline{\mathbf{p}}\}\}, \quad \text{for atomic } p. \\
T(\neg\phi) = F(\phi) & F(\neg\phi) = T(\phi). \\
T(\phi \wedge \psi) = T(\phi) \otimes T(\psi) & F(\phi \wedge \psi) = F(\phi) \cup F(\psi), \\
T(\phi \vee \psi) = T(\phi) \cup T(\psi) & F(\phi \vee \psi) = F(\phi) \otimes F(\psi). \\
T(\forall x \phi) = \bigotimes_{d \in D} T\left(\phi\left[x/d\right]\right) & F(\forall x \phi) = \bigcup_{d \in D} F\left(\phi\left[x/d\right]\right). \\
T(\exists x \phi) = \bigcup_{d \in D} T(\phi[x/d]) & F(\exists x \phi) = \bigotimes_{d \in D} F(\phi[x/d]).
\end{array}
$$

$$T(\phi) \otimes T(\psi) = \{A \cup B \mid A \in T(\phi), B \in T(\psi)\}$$

Some examples

$$T(p) = \{\{\mathbf{p}\}\}, \quad T(\neg p) = \{\{\overline{\mathbf{p}}\}\} \quad T(p \wedge q) = \{\{\mathbf{p}, \mathbf{q}\}\}$$

$$T(p \vee q) = \{\{\mathbf{p}\}, \{\mathbf{q}\}\} \quad T(p \vee q \vee (p \wedge q) = \{\{\mathbf{p}\}, \{\mathbf{q}\}, \{\mathbf{p}, \mathbf{q}\}\}$$

$$T(p \to q) = T(\neg p \vee q) = \{\{\overline{\mathbf{p}}\}, \{\mathbf{q}\}\}$$

$$T((p \vee q) \wedge (r \vee s)) = \{\{\mathbf{p}, \mathbf{r}\}, \{\mathbf{p}, \mathbf{s}\}, \{\mathbf{q}, \mathbf{r}\}, \{\mathbf{q}, \mathbf{s}\}\}$$