Future Contingents

Marco Degano

Philosophical Logic 2024
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Readings

Optional:

- Prior, A. N. (1953). ‘Three-valued logic and future contingents’. *The Philosophical Quarterly*, 317-326.
- Belnap, N., Perloff, M., & Xu, M. (2001). *Facing the future: agents and choices in our indeterminist world.* Oxford University Press.
- MacFarlane, J. (2003). ‘Future contingents and relative truth’. *The Philosophical Quarterly*, 53(212), 321-336.
- van Benthem, Johan (1991). *The logic of time*. Boston: Kluwer Academic Publishers.

Plan

1. Future Contingents
2. Łukasiewicz three-valued logic
3. Supervaluationism
4. Temporal Logics

Outline

Future Contingents

Future contingents are **contingent** statements about the **future**.

(1) It will rain tomorrow.

Contingent: neither necessarily true nor necessarily false. (may or may not occur)

Future: future (what is not yet determined?)

Can you think of statements about the future which are necessary?

Future contingents: Sea-battle

Aristotle's famous example:

(2) Tomorrow, there will be a sea-battle.

(3) Tomorrow, there will not be a sea-battle.

Is (2) true or false? What about (3)?

Sea-battle

If we take the sea-battle as contingent (i.e., it is neither necessary nor impossible), then we may consider statements like (2) or (3) as neither true nor false.

Now consider:

(4) There will be a sea battle tomorrow or there will not be a sea battle tomorrow.

Is (4) contingent?

Aristotle's claim: (4) is necessarily true.

Questions

Are future contingents true, false, or indeterminate?

Should the law of excluded middle apply to future contingents?

Is the future determined or not-determined?

Does the past/present/future exist?

Outline

Łukasiewicz three-valued logic Ł3

We have already encountered Ł3, a three-valued logic proposed by Jan Łukasiewicz in the 1920s.

$\wedge$	1	i	0
1	1	i	0
i	i	i	0
0	1	0	0

$\vee$	1	i	0
1	1	1	1
i	1	i	i
0	1	i	0

$\rightarrow$	1	i	0
1	1	i	0
i	1	$\mathbf{1}$	i
0	1	1	1

$\neg$	
1	0
i	i
0	1

Future contingents statements receive the value i.

Modalities in Ł3

In Ł3 we can define modalities as follows:

$\Diamond\phi := \neg\phi \to \phi$

$\Box\phi := \neg\Diamond\neg\phi = \neg(\phi \to \neg\phi)$

p	$\neg p \to p$	$\neg(p \to \neg p)$	$\Diamond p \wedge \neg\Box p$
1	1	1	0
i	1	0	1
0	0	0	0

However, this results in a quite peculiar modal system. For instance, we get that $\Diamond$ distributes over $\wedge$:

$$\Diamond\phi \wedge \Diamond\psi \text{ iff } \Diamond(\phi \wedge \psi)$$

Exercise

Exercise: Can you define an array of 'modal' operators $\Diamond_k$ for n-valued logics s.t. $\Diamond_k(\phi)$ iff $v(\phi) \geq \frac{k}{n-1}$ for $0 \leq k \leq n-1$?

Which operators $\Diamond_k$ can you define in the basic language so far considered? Which not?

Consider adding a unary connective N s.t. $N(\phi) = i$ for any possible value of $v(\phi)$.

Which operators $\Diamond_k$ can you define now? How?

Problems (1)

Certain statements are necessarily true.

(5) a. If there will be a sea-battle tomorrow, then there will be a sea-battle tomorrow.
b. $\phi \rightarrow \phi$

But we cannot account for (6-a), as (6-b) is not necessary (it is not valid in Ł3).

(6) a. There will be a sea-battle tomorrow OR there will not be a sea-battle tomorrow
b. $\phi \vee \neg\phi$

Problems (2)

There should be a difference between (7) and (8).

(7) There will be the Apocalypse tomorrow.

(8) The sun will rise tomorrow.

Fuzzy logic approach with different degrees of indeterminacy?

Still, the truth of the disjunction between (8) and its negation is not accounted.

Outline

Supervaluationism

We know the idea: truth is supertruth (in all valuations, formulas receive value 1).

We can view valuations as possible futures.

A future statement is true if it is true in all possible futures.

A future statement is false if it is true in all possible futures.

A future statement is contingent if it is true in some possible futures and false in others.

But what is a possible future? Let's make this more formal.

Outline

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2. Łukasiewicz three-valued logic
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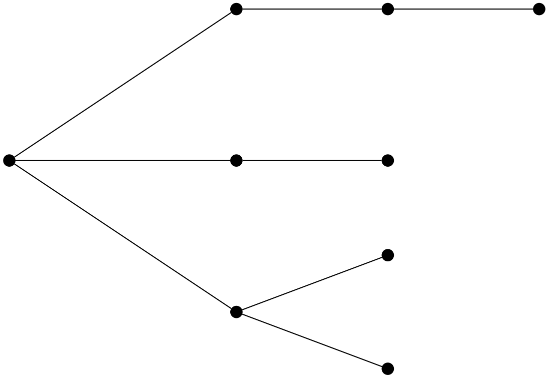
Temporal Logic

We can represent moments/times by means of a tree-like structure:



To simplify, the leftmost point is the actual time. **Histories** correspond to the (maximal) paths in the tree.

Arguably, the past is linear. What about the future?

Temporal Structures

We can represent temporal structures by means of $\langle Mo, \leq \rangle$, where Mo is a set of moments and $\leq$ a partial order on Mo.

We can define a set of histories H as the maximally ordered linear subsets of $\langle Mo, \leq \rangle$.

One can define $<$ based on $\leq$, impose certain constraints on $<$ and Mo, and study the resulting structure (van Benthem 1991).

Temporal vs Branching

Linear temporal logics assume that no branching is possible. Time is linear.

Branching temporal logics assume that branching is possible in the future, but no backward branching.

Temporal Logic

We can interpret formulas in models M with $\langle Mo, \leq \rangle$ equipped with a valuation function V, relative to a time t and a history h. We use $H(t) = \{h : t \in h\}$ for the set of histories through t.

$M, t, h \models p$	iff	$V(p, t) = 1$
$M, t, h \models \phi \wedge \psi$	iff	$M, t, h \models \phi$ and $M, t, h \models \psi$
$M, t, h \models \neg\phi$	iff	$M, t, h \not\models \phi$
$M, t, h \models F\phi$	iff	$M, t', h \models \phi$ for some $t' \in h$ and $t < t'$
$M, t, h \models P\phi$	iff	$M, t', h \models \phi$ for some $t' \in h$ and $t' < t$
$M, t, h \models \Diamond\phi$	iff	$M, t, h' \models \phi$ for some $h' \in H(t)$

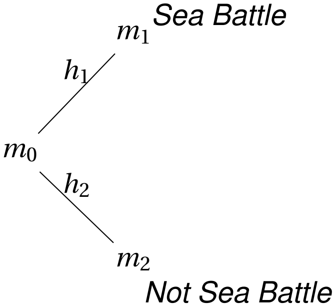
Given a history h, a formula ϕ is h-valid in a temporal model M (denoted $M, h \models \phi$) iff it is true at every time instant in that history and in that model.

Given a history h, a formula ϕ is valid in a temporal model M (denoted $M \models \phi$) iff it is true at every history and every time instant (relative to the history) in that model.

Returning to the sea battle: branching

Branching: there are many futures.

Indeterminism: each future is **metaphysically possible.**



Prior Necessity Future

Prior (1967) views future statements as 'necessary': true in all possible futures. He defines the clause for $F\phi$ as follows:

$$M, t, h \models F\phi \quad \text{iff} \quad \forall h' \in H(t), \exists t' \in h' \text{ s.t. } t < t' \text{ and } M, t', h' \models \phi$$

The result is that future contingents are just false.

More in general, the statement below is false as well[1]:

(9) a. There will be a sea-battle tomorrow, or there will not be a sea battle tomorrow.

b. $F(\phi) \vee F(\neg\phi)$

[1] By fixing a time t_i for 'tomorrow' (e.g., as a distance from the actual time)

Possible Future

We can also define a notion of 'possible' future.[2]

$M, t, h \models f\phi$ iff $\exists h' \in H(t), \exists t' \in h'$ s.t. $t < t'$ and $M, t', h' \models \phi$

[2] Based on this, one can define the usual 'always' operator in temporal logic as $\neg f \neg \phi$, which is not equivalent to the necessary future $F\phi$.

Some Problems

(10) a. There will be a sea battle tomorrow.
b. There will be a sea battle tomorrow, or there will not be a sea battle tomorrow.

What is the force of the future in (10)? If it is 'possible', why an explicit adverb like 'perhaps' or 'possibly' is not present?

(11) a. Tomorrow, there will not be a sea battle.
$F\neg p$
b. It is not the case that there will be a sea battle tomorrow.
$\neg Fp$

(11-a) and (11-b) are not equivalent under this treatment of F, arguably contrasting our intuitions.

Supervaluationist

According to a supervaluationist analysis, we have that ϕ is supertrue in moment t iff ϕ is true at every history h passing through t:

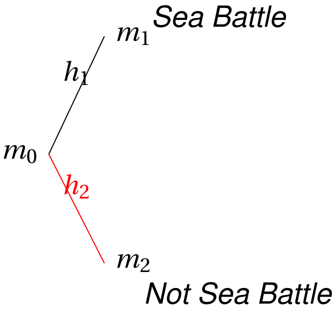
$$M, t \models \phi \quad \text{iff} \quad \forall h \in H(t) : M, t, h \models \phi$$

Future contingents are neither supertrue nor superfalse in this sense.

The Thin Red Line

Determinism: there is only one future which is metaphysically possible, the actual one.

There are many futures which are epistemically possible.



The Red Line

What constraints on the red line, the history r.

Arguably, for the current time t, $t \in r$.

Do we also want to assume that if $t_1 < t_2$ and $t_1 \in r$, then $t_2 \in r$?

This would imply that no branching is possible! Indeterminism is lost.

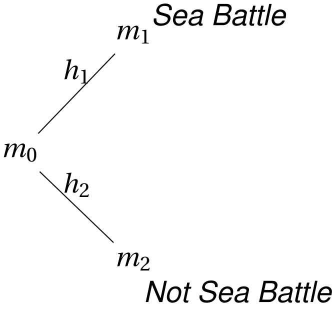
In general, there is no clear metaphysical grounding for future-directed facts in the current physical reality. The thin red line seems to posit facts about the future that cannot be explained by the present state of affairs.

MacFarlane Intuition

MacFarlane (2003): both accounts are possible, depending on the **context of assessment**.

From the point of view of m_0: 'There will be a sea-battle tomorrow' is neither true nor false.

From the point of view of m_1 or m_2: 'There will be a sea-battle tomorrow' is determinately true (or false).



MacFarlane Relativism

Distinction between moment of utterance and moment of assessment.

ϕ is true at a context of utterance t and context of assessment a:

$$M, t, a \models \phi \quad \text{iff} \quad \forall h \text{ s.t. } h \in H(t) \;\&\; h \in H(a) : M, t, h \models \phi$$

Let p be ‘there will be a sea battle’.

For $t = m_0$ and $a = m_0$, p is neither true nor false.

For $t = m_0$ and $a = m_1$, p is false.

For $t = m_0$ and $a = m_2$, p is true.

Future Contingents and Beyond

Can other frameworks which we encountered during the course be applied to future contingents?

Logic of Paradox: a statement like ‘It will rain tomorrow’ can be assigned both true and false truth values at present because its future resolution is undecided. The future’s openness manifests in logical paradoxes at the present.

Probability: ‘It will rain tomorrow’ is assigned a probability, updated dynamically based on new information. Each branch could be assigned a probability corresponding to its plausibility.

Conditionals: Conditional reasoning identifies the specific conditions under which certain branches would become actual. In general, how to analyse conditionals related to the future?

A final example

There will be an exam next week, and you are going to do well.

While contingent, let's posit a 'thin red line' where this fact becomes true!

THANK YOU!