# Assignment 1

Philosophical Logic 2024/2025

## Instructions

- Discussion among students is allowed, but the assignments should be done and written individually.
- Late submissions will be accepted until three days after the deadline, with a 0.5 penalty per day.
- Please be explicit and precise, and structure your answers in a way that makes them easy to follow.
- For induction proofs, one or two cases besides the basic step are usually enough. Always include the case of $\rightarrow$ if present in the language. If you feel safer, you can include the full induction.
- Please submit your answers as PDF and use *PL-2024-A1-⟨your-last-name⟩* as the name of your file.
- For any questions or comments, please contact us at m.degano@uva.nl, s.b.knudstorp@uva.nl, fatima.scha@student.uva.nl.
- **Deadline: Wednesday 6 November 2024, 9 pm**

## Exercise 1 [20 points]

Choose or invent a paradox that fascinates you. Provide a clear and concise outline of the paradox, including the assumptions and the conclusion that lead to the paradoxical outcome. Explain briefly why you find this paradox interesting or significant. Offer a brief explanation of how you might resolve the paradox, or explain why it cannot be solved in your view. Be creative in your presentation: you may choose to illustrate your paradox visually, describe it textually, …

## Exercise 2 [25 points]

Determine whether the following hold or not in classical logic $CL$, Strong Kleene $K_3^s$, Weak Kleene $K_3^w$ Łukasiewicz Ł3, Logic of Paradox $LP$ and $RM3$ logic.

1. $\vDash \neg(\neg p \rightarrow p)$
2. $p \rightarrow q, q \rightarrow r \vDash p \rightarrow r$
3. $\vDash ((p \rightarrow q) \rightarrow p) \rightarrow p$
4. $\neg p, p \vee q \vDash q$
5. $q, \neg p \rightarrow \neg q \vDash p$

You do not need to show your workings, but simply state your answers, as in the following template:

| | $CL$ | $K_3^s$ | $K_3^w$ | Ł3 | $LP$ | $RM3$ |
|---|---|---|---|---|---|---|
| $\vDash p \rightarrow p$ | yes | no | no | yes | yes | yes |

## Exercise 3 [25 points]

We have seen that in $K_3^s$ and $K_3^w$, the implication $\to$ is definable using $\vee$ and $\neg$, as $p \to q \equiv \neg p \vee q$. In Ł3 we have that $p \to q \not\equiv \neg p \vee q$.

Furthermore, in Ł3 we cannot express $p \to q$ using only $\neg$, $\vee$ and $\wedge$. Prove this fact: that is, prove that for any formula $\phi$ whose only sentence letters are $p$ and $q$ and has no other connective besides $\neg$, $\vee$ and $\wedge$, there is a valuation $v$ s.t. $v(\phi) \neq v(p \to q)$.

## Exercise 4 [30 points]

For each $n \in \mathrm{N} \geq 2$ let Ł$n$ be the $n$-many valued logic constructed from the set of truth values $\{0, 1/(n-1), 2/(n-1), \dots, 1\}$ (i.e., for any $n \in \mathrm{N} \geq 2$, $T_n = \{k/(n-1) \mid 0 \leq k \leq n-1, k \in \mathrm{N}\}$) and with the following semantic clauses. In particular, for $n = 3$ we get the semantic clauses of our usual Łukasiewicz three-valued logic.

$$
\begin{aligned}
v(\neg\phi) &= 1 - v(\phi) \\
v(\phi \wedge \psi) &= \min(v(\phi), v(\psi)) \\
v(\phi \vee \psi) &= \max(v(\phi), v(\psi)) \\
v(\phi \to \psi) &= \begin{cases} 1 & \text{if } v(\phi) \leq v(\psi) \\ 1 - (v(\phi) - v(\psi)) & \text{otherwise} \end{cases}
\end{aligned}
$$

Let the consequence relation of each $n$-valued logic be $\models_n$, defined as follows.

$$\Gamma \models_n \phi \text{ iff for any } n\text{-valued valuation } v, \text{ if } v(\gamma) = 1 \text{ for all } \gamma \in \Gamma, \text{ then } v(\phi) = 1.$$

Do (1), (2), (3) and (4) below hold? For each of these, if it holds, prove it. If it does not, provide a counterexample.

(1) $\models_3 \phi \Rightarrow \models_4 \phi$

(2) $\models_4 \phi \Rightarrow \models_3 \phi$

(3) $\models_3 \phi \Rightarrow \models_5 \phi$

(4) $\models_5 \phi \Rightarrow \models_3 \phi$