Final Exam

Philosophical Logic 2024/2025

Exercise 1 [30 points]

Show that

1. **Fuzzy Logic Ł$_c$:** $p \to (p \to q) \nvDash_{Ł_c} p \to q$
2. **Counterfactuals:** $p \rightsquigarrow q \nvDash \neg q \rightsquigarrow \neg p$
3. **Supervaluations** (global consequence relation):
   the following meta-inference (reductio) fails:
   if $\phi \wedge \psi \vDash_g \chi$ and $\phi \wedge \psi \vDash_g \neg\chi$, then $\vDash_g \neg(\phi \wedge \psi)$.
   (i.e., find formulas ϕ, ψ, χ s.t. $\phi \wedge \psi \vDash_g \chi$ and $\phi \wedge \psi \vDash_g \neg\chi$ but $\nvDash_g \neg(\phi \wedge \psi)$)
   Hint: $\phi \vDash_g \Delta\phi$
4. **Non-monotonic logic:** the following rule is derivable in **P**:
   if $\phi \wedge \psi \mathrel{|\!\sim} \chi$, then $\phi \mathrel{|\!\sim} \psi \supset \chi$,
   where $\supset$ is the material conditional $\phi \supset \psi \equiv \neg\phi \vee \psi$
   (i.e., you need to provide a proof-theoretic derivation of $\phi \mathrel{|\!\sim} \psi \supset \chi$ in **P** taking $\phi \wedge \psi \mathrel{|\!\sim} \chi$ as an additional axiom; you are not allowed to use completeness of plausible consequence)
5. **Probability:** $p \to r, q \to r \vDash_P (p \vee q) \to r$, where '$\to$' is the indicative conditional, defined using conditional probability $P(\phi \to \psi) = P(\psi|\phi) = \frac{P(\psi \wedge \phi)}{P(\phi)}$.

   Tip: (4) and (5) are less immediate than (1), (2) and (3). If you struggle, move to the next exercises, and return to Exercise 1 later.

Exercise 2 [25 points]

Consider the Weak Kleene K_3^w three-valued logic and prove the following, where $\vDash_{CL}$ is the classical logic consequence relation:

(a) Show that if (i) $\phi \vDash_{CL} \psi$ and (ii) every sentence letter occurring in ψ occurs in ϕ, then $\phi \vDash_{K_3^w} \psi$.

Hint: Prove and use that for all formulas ϕ and three-valued valuations v: $v_{K_3^w}(\phi) = i$ iff there is a sentence letter p occurring in ϕ s.t. $v(p) = i$.

(b) Show that the converse fails: there are formulas ϕ, ψ s.t. $\phi \vDash_{K_3^w} \psi$, but it is not the case that both (i) $\phi \vDash_{CL} \psi$ and (ii) every sentence letter occurring in ψ occurs in ϕ.

Hint: if $\phi \vDash_{K_3^w} \psi$, then $\phi \vDash_{CL} \psi$.

Exercise 3 [20 points]

Exercise 3 concerns truthmaker semantics (see Definitions).

(a) Show that for all formulas ϕ, ψ, χ:

(1) $\phi \wedge (\psi \vee \chi) \vDash_{TM} (\phi \wedge \psi) \vee (\phi \wedge \chi)$

(2) $(\phi \wedge \psi) \vee (\phi \wedge \chi) \vDash_{TM} \phi \wedge (\psi \vee \chi)$

(b) (1) Show that there are no tautologies in truthmaker semantics (i.e., there is no formula ϕ s.t. for all models M and states $s \in M$: $s \vDash^+ \phi$).

(2) Show that every set of formulas is satisfiable (i.e., for every set of formulas Γ, there is a model M and $s \in M$ s.t. for all $\gamma \in \Gamma$: $s \vDash^+ \gamma$).

Hint for both (1) and (2) of (b): this is *simpler* than it looks.

Exercise 4 [25 points]

According to fuzzy logic, truth comes in degrees. Consider the pair of sentences below. Why are such examples problematic for a degree-theoretic perspective? How can fuzzy theorists reply to the challenge?

1. If John is tall, then John is tall.
2. If John is tall, then John is not tall.