Assignment 3

Philosophical Logic 2024/2025

Instructions

- Discussion among students is allowed, but the assignments should be done and written individually.
- Late submissions will be accepted until three days after the deadline, with a 0.5 penalty per day.
- Please be explicit and precise, and structure your answers in a way that makes them easy to follow.
- For induction proofs, one or two cases besides the basic step are usually enough. Always include the case of $\rightarrow$ if present in the language. If you feel safer, you can include the full induction.
- Please submit your answers as PDF and use *PL-2024-A3-⟨your-last-name⟩* as the name of your file.
- For any questions or comments, please contact {m.degano, s.b.knudstorp, f.scha}@uva.nl
- **Deadline: Friday 22 November 2024, 9 pm**

Exercise 1 [20 points]

John, at time x, utters the sentence:

(1) 'What John says at time x is not true.'

This statement is recognized as paradoxical due to its self-referential nature. Thus Bob, at a later time, says:

(2) 'What John says at time x is not true.'

Although (1) and (2) are identical in form, they seem to be judged differently — (1) is seen as paradoxical, while (2) is not.

Do you agree with the judgment that (1) is paradoxical and (2) is not? Despite being the same sentence, why might (1) and (2) be judged differently? Is it possible to assign a truth value to (2)? Why or why not?

Use no more than 250 words in your answer.

Exercise 2 [20 points]

In class, we examined how the liar sentence $\psi = \neg T(`\psi')$ leads to a contradiction $\perp$ using the T-in and T-out rules:

T-in: $\vDash \phi \rightarrow T(`\phi')$
T-out: $\vDash T(`\phi') \rightarrow \phi$

T-enter: if $\vDash \phi$ then $\vDash T(`\phi')$
T-exit: if $\vDash T(`\phi')$ then $\vDash \phi$

Show how you can derive $\perp$ using only T-out and T-enter, which is weaker than T-in (i.e., you are not allowed to use T-in). You can use LEM, explosion, T-enter, T-out, the meaning of ψ, proof by cases, $\wedge$-Intro.

(Note that $\vDash \phi$ means that ϕ is proven true without assumptions.)

Exercise 3 [35 points]

Consider the Kripke's theory of truth presented in the slides.

1. Let $M = \langle D, I, \mathcal{T} \rangle$ and $M' = \langle D, I, \mathcal{T}' \rangle$ be two models such that $\mathcal{T} \subseteq \mathcal{T}'$. Prove that $J(\mathcal{T}) \subseteq J\left(\mathcal{T}'\right)$. (slide 26)
2. Let $\mathcal{T}$ and $\mathcal{T}'$ be coherent and $\mathcal{T} \subseteq \mathcal{T}'$. Prove that $\mathcal{T}^* \subseteq \mathcal{T}'^*$ (slide 30)
3. Find a formula ϕ which is false in some but not all fixed points and true in no fixed point. Motivate your answer. (slide 31)
4. We have seen how we can construct the revenge paradox by adding $\sim$ to the language. (slide 32)

   Suppose that, instead of adding a new operator to the language, we change the original clause of $\to$ as follows:

   $M \vDash \phi \to \psi$ iff $M \dashv \phi$ or $M \vDash \psi$ or ($M \nvDash \phi$ and $M \nvDash \psi$ and $M \not\dashv \phi$ and $M \not\dashv \psi$)
   $M \dashv \phi \to \psi$ iff $M \vDash \phi$ and $M \dashv \psi$

   Can we generate a revenge paradox with the new semantics for $\to$? How?

Exercise 4 [25 points]

Show that the logic of ST (ST) has the same consequence relation of classical logic (CL).

You need to show that

for any set of propositional formulas Γ and any propositional formula ϕ, we have $\Gamma \vDash_{ST} \phi$ iff $\Gamma \vDash_{CL} \phi$.

In particular, you can consider the logic of ST as a three-valued propositional logic with $\neg, \vee, \wedge, \to$ as connectives with a Strong Kleene semantics, but with the following notion of logical consequence:

Given a set of formulas Γ and a formula ϕ, we say that Γ entails ϕ and we write $\Gamma \vDash_{ST} \phi$ iff for any valuation v s.t. $v(\gamma) \in \{1\}$ for all $\gamma \in \Gamma$, then $v(\phi) \in \{1, i\}$.

This exercise is presented as exercise 3.7 and 3.8 in the lecture notes, and you are welcome to follow the steps mentioned there. If you rely on the facts mentioned in exercise 3.7, you need to prove them.

ST is based on Strong Kleene semantics. While keeping the same notion of logical consequence of ST, we consider the following two variants and whether the statement above holds: (i) Weak Kleene semantics; (ii) Łukasiewicz semantics. For each of these, if it holds, motivate your answer (no full proofs needed). If it does not, provide a counterexample.