Vagueness IV

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Readings

Optional:

- van Rooij, Robert (2011). *Vagueness and linguistics*. Vagueness: A guide. Dordrecht: Springer Netherlands. pp. 123-170.

Outline

Contextualism

Contextualism endorses the view that vague predicates are context-dependent or context-sensitive.

The inductive hypothesis (or the set of conditional premises) is false, but it is intuitively appealing. Contextualist solutions try to account for this.

Indistinguishability Relation $\sim$

We can state the premise of the Sorites as follows:

If we deem one individual x tall, and this individual is indistinguishably taller than another individual y, then we must deem y tall as well.

More formally, with $x \sim_P y$ as 'x is indistinguishable from y':

$$\text{for any } x, y \in D, \ (Px \wedge x \sim_P y \rightarrow Py)$$

What kind of relation should $\sim_P$ be? Can it be transitive?

Indistinguishability Relation $\sim$

We define $x \sim_P y := \neg x \succ_P y \wedge \neg y \succ_P y$

$x \succ_P y$ as x is significantly P-er than y.

$x \sim_P y$ as there is no significant difference between x and y

What kind of ordering should $\succ_P$ be?

If $\succ_P$ is a strict weak order (irreflexive, transitive and almost connected), then $\sim_P$ results in an equivalence relation (hence, transitive).

Semi orders

We define $\succ_P$ as a **semi-order**:

Irreflexive:
$\forall x : \neg x \succ x$
Interval-order:
$\forall x, y, v, w : (x \succ y \wedge v \succ w) \rightarrow (x \succ w \vee v \succ y)$
Semi-transitive
$\forall x, y, z, v : (x \succ y \wedge y \succ z) \rightarrow (x \succ v \vee v \succ z)$

Here $\sim_P$ is reflexive and symmetric, but need not be transitive.

Context-dependent $\sim$

Contextualist solution: $\sim_P$ is context-dependent and the context changes in a Sorites sequence.

Similarity depends on a contextually given comparison class:

$x \sim_P^c y$ iff $\neg \exists z \in c : x \sim_P z \not\sim_P y$ or $x \not\sim_P z \sim_P y$

x and y are similar wrt the comparison class c if x and y are not (even) indirectly distinguishable w.r.t. elements of c.

Context-dependent $\sim$

If we look at conditionals in isolation, we do not run into problems:

$$(P(x, c) \wedge x \sim_P^c y) \to P(y, c)$$

Here c consists just of $\{x, y\}$

But we cannot consider all the premises together:

1. $P(x, c)$ with $c = \{x, y, z\}$
2. $(P(x, c) \wedge x \sim_P^c y) \to P(y, c)$ with $c = \{x, y\}$
3. $(P(y, c) \wedge y \sim_P^c z) \to P(z, c)$ with $c = \{y, z\}$
4. $P(z, c)$ with $c = \{x, y, z\}$

From (1)–(3), we cannot derive (4).

Contextualism - Experimental Evidence

What kind of test would support contextualism?

Forced march experiment: a situation where one is asked, step by step, whether a property like ‘heap’ or ‘bald’ still applies after a slight change, such as removing a grain of sand or a hair.

Different presentations of the stimuli, more colour variation could also potentially lead to different results.

But still vague predicates can lead to paradoxical conclusions in a broader sense, without involving any explicit sequence of verbal responses.

Outline

The Sorites

$$\begin{array}{l} \phi(1) \\ \phi(1) \rightarrow \phi(2) \\ \phi(2) \rightarrow \phi(3) \\ \ldots \\ \phi(1M-1) \rightarrow \phi(1M) \\ \hline \phi(1M) \end{array}$$

Reject the conditional premises (1)

Three-valued logic reply (Strong Kleene): some of the conditionals premises receive the value i

But

(i) all the conditionals feel true, rather than indeterminate;

(ii) arbitrary boundary;

(iii) higher-order vagueness.

Reject the conditional premises (2)

Fuzzy logic reply (logical consequence as truth preservation): the premises are ‘almost true’, but not ‘fully true’;

(i) arbitrary boundary/artificial precision (unnatural mathematical precision on inherently vague concepts);

(ii) compositionality of truth degree;

(iii) higher-order vagueness (why is 0.6 assigned to 15 hairs and not 0.55 or 0.65?);

(iv) tolerance is not respected.

Reject the conditional premises (3)

Supervaluationism reply: some of the conditionals are not supertrue, but this does not imply that they are superfalse (i.e., we are not committed to a boundary)

(i) artificial precision (precisifications draw exact lines for vague predicates)

(ii) higher-order vagueness (your assignment)

(iii) if Δ is added to the logic, some important meta theorems are lost

Reject the conditional premises (4)

Epistemic reply: one of the premises is false, but we do not know which one.

(i) counterintuitive sharp boundaries;

(ii) semantic competence on these boundaries;

(iii) tolerance is not addressed.

Reject the conditional premises (5)

Contextualist reply: only a weakened version of the premises is valid

(i) it needs to be supplemented with a formal theory to test all the predictions.

Reject the validity of the argument (1)

Three-valued reply (Logic of Paradox): modus pones fails

(i) counterintuitive nature of borderline cases (gluts and not gaps);

(ii) inferences rules are not preserved (e.g., explosion);

(iii) higher order-vagueness (Sorites series still fine with Δ operator).

Reject the validity of the argument (2)

Fuzzy logic reply (logical consequence as degree preservation): modus ponens fails;

(i) the critical remarks for truth preservation fuzzy logics remain.

Reject the validity of the argument (3)

Subvaluationism reply: modus ponens fails

(i) the critical remarks for logic of paradox remain;

(ii) [quite unnatural proof-theoretic system].