Assignment 2

Philosophical Logic 2024/2025

Instructions

- Discussion among students is allowed, but the assignments should be done and written individually.
- Late submissions will be accepted until three days after the deadline, with a 0.5 penalty per day.
- Please be explicit and precise, and structure your answers in a way that makes them easy to follow.
- Please submit your answers as PDF and use *PL-2024-A2-⟨your-last-name⟩* as the name of your file.
- For any questions or comments, please contact {m.degano, s.b.knudstorp, f.scha}@uva.nl
- **Deadline: Wednesday 13 November 2024, 9 pm**

Exercise 1 [25 points]

On one view, vagueness can be attributed to our lack of knowledge or the precision of our language in describing the world. On a different view, however, vagueness exists in the world itself - meaning that some things are inherently indeterminate.

For example, consider a vague object like a cloud. Its boundaries might be unclear, not because we lack sufficient knowledge about them, but because the boundaries themselves are objectively indeterminate.

(i) Do you find the idea that vagueness exists in the world itself plausible? Why or why not?

(ii) One might argue that vagueness can also be applied also to moral or metaphysical cases (e.g., the sorites paradox and the moral status of a fetus during pregnancy; the gradual loss of identity in someone experiencing severe cognitive decline, ...)? Are these also cases of vagueness-in-the-world? Do you find an appeal to vagueness (of any kind) satisfactory in addressing these kinds of moral or metaphysical cases? Why or why not?

Use no more than 400 words in your answer.

Exercise 2 [25 points]

Consider the fixed marginal models $M = \langle W, d, \alpha, I \rangle$ used by Williamson to model inexact knowledge. Please have a look at the definitions on Canvas. Recall informally that $\Box\phi$ stands for inexact knowledge (i.e., knowledge within the margin α):

$M, x \vDash \Box\phi$ iff $\forall y$ s.t. $d(x, y) \leq \alpha$, $M, y \vDash \phi$

(a) For this part of the exercise, we work with the concrete and simplified model discussed in class. In particular, we want to model a situation like the Sorites by having a model $W = \{n | 0 \leq n \leq 10\}$, $\alpha = 1$, $d(k, l) = |k - l|$ for any $k, l \in W$, and $I(p) = \{m | 0 \leq m \leq 4\}$.

Intuitively, at 0 we are absolutely sure that p holds, and we might want to express this by requiring that $\Box^n p$ for any finite n. Show that, however, for some n, $\Box^n p$ does not hold at 0.

In general, in fixed marginal models, (1) is valid (true in every model everywhere). But (2) and (3) are not.

(1) $\Box\phi \to \phi$ — *If I know ϕ, then ϕ is true*

(2) $\Box\phi \to \Box\Box\phi$ — *If I know ϕ, then I know that I know ϕ.*

(3) $\neg\Box\phi \to \Box\neg\Box\phi$ — *If I don't know ϕ, then I know that I don't know ϕ.*

(b) For each of (2) and (3) above, assume that the principle is valid. Can we still faithfully represent situations like the Sorites? Motivate your answer. You need to discuss two cases separately: one in which (2) is valid, and one in which (3) is valid.

(c) Consider the principles in (2) and (3) from a broader perspective. Do you find them plausible? Should there be a difference in contexts of inexact knowledge versus exact knowledge?

Use no more than 200 words for part (c).

Exercise 3 [50 points]

Background

Consider a modalized version of supervaluationism with a definitely operator Δ. Please have a look at the definitions on Canvas. This exercise explores higher-order vagueness, its potential issues, and different notions of logical consequence.

Part 1

Consider John, who is a borderline case of being tall, and let q be "John is tall". We can represent borderline cases as:

$$\neg\Delta q \wedge \neg\Delta\neg q$$

(This states that it's neither definitely the case that John is tall, nor definitely the case that he isn't tall.)

Since "definitely tall" is itself vague, we are faced with higher-order vagueness:

$$(1) \quad \neg\Delta\Delta q \wedge \neg\Delta\neg\Delta q$$

Part 2

Consider a sorites series from 1 to m from tall to not-tall. We know that 1 is tall and thus p_1, while m is not tall and thus $\neg p_m$. Due to the presence of borderline cases, supervaluationism commits to:

$$(2) \quad \Delta p_i \to \neg\Delta\neg p_{i+1}$$

(2) states that there can be no sharp transitions from definitely tall to definitely not tall.

Higher-order vagueness implies we cannot find sharp transitions at any order, leading to principles of the form $\Delta\Delta^n p_i \to \neg\Delta\neg\Delta^n p_{i+1}$.

In particular, in the rest of this exercise we consider the case where $m = 5$, and we take the following principles as valid:

(i) $\Delta^4 p_1 \to \neg\Delta\neg\Delta^3 p_2$

(ii) $\Delta^3 p_2 \to \neg\Delta\neg\Delta^2 p_3$

(iii) $\Delta^2 p_3 \to \neg\Delta\neg\Delta p_4$

(iv) $\Delta p_4 \to \neg\Delta\neg p_5$

Questions

Using **global logical consequence** $\vDash_g$ (preservation of supertruth):

1.a Is supervaluationism compatible with higher-order vagueness? That is, is there a non-empty set of valuations V such that $V \vDash^{!1}$ (1)?

2.a Show that by accepting (i), (ii), (iii), (iv) above, you can derive a contradiction (e.g., $p_m \wedge \neg p_m$).

Recall that we are modelling a Sorites series from 1 to 5. Thus, we are working with models where $V \vDash^{!1} p_1$ and $V \vDash^{!1} \neg p_5$.

Hint: Recall that $\phi \vDash_g \Delta\phi$

Alternative Framework

Now consider a different notion of supertruth (admissible supertruth) based on "admitted" valuations from a single point of evaluation. Let R be an 'admissibility' accessibility relation over a non-empty set of valuations V. The Δ operator is also governed by R and the accessibility relation is not universal anymore.

$$
\begin{array}{lll}
(V,R), v \vDash p & \text{iff} & v(p) = 1 \\
(V,R), v \vDash \neg\phi & \text{iff} & (V,R), v \nvDash \phi \\
(V,R), v \vDash \phi \wedge \psi & \text{iff} & (V,R), v \vDash \phi \text{ and } (V,R), v \vDash \psi \\
(V,R), v \vDash \phi \vee \psi & \text{iff} & (V,R), v \vDash \phi \text{ or } (V,R), v \vDash \psi \\
(V,R), v \vDash \phi \rightarrow \psi & \text{iff} & (V,R), v \nvDash \phi \text{ or } (V,R), v \vDash \psi \\
(V,R), v \vDash \Delta\phi & \text{iff} & \forall v' \in V \text{ such that } vRv' : V, v' \vDash \phi
\end{array}
$$

Truth is admissible supertruth: $(V,R) \vDash \phi$ iff for any $v \in V : (V,R), v' \vDash \phi$ **for every v' s.t. vRv'**.

Logical consequence: $\Gamma \vDash \phi$ iff for any (V,R) and any $v \in V$, if $(V,R), v' \vDash \phi$ **for every v' s.t. vRv'**, then $(V,R), v' \vDash \psi$ **for every v' s.t. vRv'**

We aim to understand which constraints on the accessibility relation R, together with the novel notion of logical consequence in the alternative framework, run or not run into the issues encountered before (where we assumed a global notion of logical consequence and a universal accessibility relation). In particular, we consider three possible cases for R:

(A) R is reflexive.

(B) R is reflexive and symmetric.

(C) R is reflexive and transitive.

Questions

For each case in (A), (B) and (C), you should discuss:

1.b Whether the issue in Part 1 is resolved or not. If yes, provide a suitable V and R s.t. $(V,R) \vDash$ (1). If not, explain why (1) is still problematic.

2.b Whether the issues in Part 2 are resolved or not.

If not, show that the issue of (2a) is still present.

If yes, provide a suitable V and R where (i), (ii), (iii) and (iv) hold without giving rise to a contradiction.