Assignment 4

Philosophical Logic 2024/2025

Instructions

- Discussion among students is allowed, but the assignments should be done and written individually.
- Late submissions will be accepted until three days after the deadline, with a 0.5 penalty per day.
- Please be explicit and precise, and structure your answers in a way that makes them easy to follow.
- Please submit your answers as PDF and use *PL-2024-A4-⟨your-last-name⟩* as the name of your file.
- For any questions or comments, please contact {m.degano, s.b.knudstorp, f.scha}@uva.nl
- **Deadline: Friday 29 November 2024, 9 pm**

Exercise 1 [20 points]

We have seen that the notion of *exact* truthmaking does not coincide with *minimal* truthmaking (slide 12 - Truthmakers II).

Can you find compelling examples of sentences/truths which have exact verifiers/truthmakers but do not have any minimal truthmakers? What about sentences/falsities which have exact falsifiers/falsemakers but do not have any minimal falsemaker? From a metaphysical perspective, would you accept the view there are truths which have exact truthmakers, but they do not have minimal truthmakers?

Use no more than 300 words

Exercise 2 [40 points]

1. Determine whether the following hold or not. If they hold, prove it (you can choose between the algorithmic procedure (slide 14 - Truthmakers I), the FDE semantics (slide 20 - Truthmakers I) or the axiomatic system (slide 18 - Truthmakers I)). If they do not hold, show this by using the algorithmic procedure (a single unsuccessful normal form is sufficient) or by exhibiting a countermodel in FDE.
   (a) $(\neg p \vee q) \wedge (\neg q \vee r) \vDash_T (\neg p \vee r)$
   (b) $(p \wedge \neg p) \vee q \vDash_T p \vee q$
2. Show that only tautological entailments, as defined in slide (14), can be proven using the axiomatic system in slide (18). You can assume that the language contains only $\wedge$, $\vee$ and $\neg$.

   To show this, show (i) that all the axioms are tautological entailments and (ii) that the rules preserve tautological entailment.

   (a) For (i), we can check this by induction on the length of the formulas. It is easy to see that for literals (propositional variables or their negation), all axioms are tautological entailments. You may assume this. Since the whole proof is tedious, in this exercise, you only need to show that the inductive step holds for the disjunction axiom below.

   Disjunction axiom: $\phi \vDash_T \phi \vee \psi$

(b) For (ii), check only the transitivity rule and the contraposition rule below. For instance, you need to check that if $\phi \vDash_T \psi$ and $\psi \vDash_T \chi$ are tautological entailments, then $\phi \vDash_T \chi$ is also a tautological entailment.

Transitivity: from $\phi \vDash_T \psi$ and $\psi \vDash_T \chi$, infer $\phi \vDash_T \chi$.

Contraposition: from $\phi \vDash_T \psi$, infer $\neg\psi \vDash_T \neg\phi$.

Recall that each $\phi \vDash_T \psi$ has a normal form $\phi_1 \vee \cdots \vee \phi_n \vDash_T \psi_1 \wedge \cdots \wedge \psi_m$ and that ϕ tautologically entails ψ iff for every ϕ_i and ψ_j, we have $\phi_i \vDash_{ET} \psi_j$ (slide 13 - Truthmakers I for the notion of $\vDash_{ET}$).

Exercise 3 [40 points]

In this exercise, we work with the truthmaker semantics defined in the lecture notes.

1. Do problem 5.c in the lecture notes
2. Do problem 5.d in the lecture notes
3. We have seen in class that (D) does not hold.

   (D) $(p \vee q) \wedge (p \vee r) \nvDash^+ p \vee (q \wedge r)$

   Consider now the following condition on truthmakers models $M = \langle S, \leq, I \rangle$

   Non-vacuity: For all propositional letters p:

   - $\exists s \in S : I^+(s, p) = 1$, AND
   - $\exists s \in S : I^-(s, p) = 1$

   (a) Does (D) hold in all non-vacuous models (i.e., for every non-vacuous model M and $s \in M$, if $s \vDash^+ (p \vee q) \wedge (p \vee r)$, then $s \vDash^+ p \vee (q \wedge r)$). If yes, prove it. If not, exhibit a countermodel.

   Consider now the following notion of truthmaking satisfaction.

   Convex truthmaking: Given a model M and $s \in M$, we define for all formulas ϕ:

   $s \vDash^{+,c} \phi$ iff there is s', s'' s.t. $s' \vDash^+ \phi$ and $s'' \vDash^+ \phi$ and $s' \leq s \leq s''$

   (b) Does (D) hold in all non-vacuous models for convex truthmaking (i.e., for every non-vacuous model M and $s \in M$, if $s \vDash^{+,c} (p \vee q) \wedge (p \vee r)$, then $s \vDash^{+,c} p \vee (q \wedge r)$). If yes, prove it. If not, exhibit a countermodel.