Vagueness II

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New Exam Date

Based on survey results, the **new exam date** is likely **Thursday, December 19, from 9:00 to 11:00 a.m**.

If you have any concerns, please contact us as soon as possible.

An announcement on Canvas will confirm both the date and the exam room.

Plan

1. Three-valued Logics and Vagueness
2. Higher-order Vagueness
3. Fuzzy Logics
4. Interlude: FDE
5. Supervaluationism

Readings

Required:

- Lecture notes: ch. 3; ch. 4.1

Optional:

- An Introduction to Non-Classical Logic (Priest): ch. 7.4, 7.10; ch. 11
- Logic for Philosophy (Sider): ch. 3.4.4-3.4.5

Outline

Back to Vagueness

Recall the structure of the Sorites paradox:

$$\begin{array}{l} \boxed{\begin{array}{l} \phi(1) \\ \phi(1) \to \phi(2) \\ \vdots \\ \phi(1M-1) \to \phi(1M) \end{array}} \\ \hline \phi(1M) \end{array} \qquad \begin{array}{l} \boxed{\begin{array}{l} \phi(1) \\ \phi(1) \to \phi(2) \\ \ldots \end{array}} \\ \boxed{\begin{array}{l} \phi(k-1) \to \phi(k) \\ \phi(k) \to \phi(k+1) \\ \ldots \\ \phi(l) \to \phi(l+1) \end{array}} \\ \boxed{\begin{array}{l} \ldots \\ \phi(1M-1) \to \phi(1M) \end{array}} \\ \hline \phi(1M) \end{array}$$

Using classical logic (left), the conclusion must be true.

Using K_3^s (right), we can make some of the premises as neither true nor false, and the conclusion false.

The logic of paradox (LP)

Can we define logical consequence differently?

Let's take the designated values to be $T^+ = \{1, i\}$.

Taking the truth-value functions of K_3^s, this leads to the so-called logic of paradox LP.

LP is a *paraconsistent* logic, as have that $p \wedge \neg p \not\models q$.

LP: gaps and gluts

In the previous three-valued systems, i is a **gap**: *neither true nor false*.

In LP, i is a **glut**: *both true and false*.

Do you find the idea of a glut plausible?

The 'Liar sentence' has value i under a LP analysis (more on this later in the course).

The logic of LP

It holds that for any formula ϕ,

$$\models_{CL} \phi \text{ iff } \models_{LP} \phi$$

Modus ponens fail:

$$p, p \to q \not\models_{LP} q$$

To fix this, we can change the truth value function for $\to$, while still keeping $T^+ = \{1, i\}$. This gives us the $RM3$ logic.

$\to$	1	i	0
1	1	0	0
i	1	i	0
0	1	1	1

[Priest 2008, ch. 7.4]

Assessing the Situation

The K_3^s answer to the sorites is: we reject some of the premises as not true

The LP answer to the sorites is: the argument is not valid (modus ponens fails)

Outline

Higher-order Vagueness

Three-valued solutions reject the idea of sharp boundaries between red and not-red (1 and 0) by introducing an additional truth value i

But then there are still boundaries between 1 and i and i and 0.

Vagueness is still there, but in a higher-order form.

Higher-order Vagueness

1: definitely red Δp

0: definitely not red $\Delta \neg p$

i: not definitely red and not definitely not red $\neg \Delta p \wedge \neg \Delta \neg p$

p	Δp
1	1
i	0
0	0

Δp take only values 1 or 0.

But the Sorites argument appears to still be problematic with sentences of the form $\Delta(\phi(n))$.

Higher-order vagueness

We specify an indefiniteness operator as

$$\nabla\phi := \neg\Delta\phi \wedge \neg\Delta\neg\phi$$

Second-order vagueness on Δp can be characterized by:

$$\neg\Delta\Delta p \wedge \neg\Delta\neg\Delta p \equiv \nabla\Delta p$$

What is the truth value of $\nabla\Delta p$?

Outline

Truth and Degrees

Truth comes in degrees. Fuzzy logics are many-valued logics where $T = \{x \in \mathcal{R} | 0 \leq x \leq 1\}$.

Connectives can be defined in different ways, leading to distinct logical systems. Here we adopt the following clauses:

$$
\begin{aligned}
v(\neg\phi) &= 1 - v(\phi) \\
v(\phi \wedge \psi) &= \min\left(v(\phi), v(\psi)\right) \\
v(\phi \vee \psi) &= \max\left(v(\phi), v(\psi)\right) \\
v(\phi \rightarrow \psi) &= \begin{cases} 1 & \text{if } v(\phi) \leq v(\psi) \\ 1 - (v(\phi) - v(\psi)) & \text{otherwise} \end{cases}
\end{aligned}
$$

What semantic clause for $v(\phi \leftrightarrow \psi)$ [i.e., $\phi \rightarrow \psi \wedge \psi \rightarrow \phi$] ?

Fuzzy Logics and Logical Consequence

Łukasiewicz logic Ł$_c$:

$\Gamma \models \psi$ iff for any $0 \leq t \leq 1$ and for any valuation v, if $v(\gamma) \geq t$ for all $\gamma \in \Gamma$, then $v(\psi) \geq t$.

Modus ponens fails! Consider $p, p \rightarrow q \models q$ with $v(p) = 0.8$, $v(q) = 0.6$

Łukasiewicz continuum-valued logic Ł$_\aleph$

$\Gamma \models \psi$ iff for any valuation v, if $v(\gamma) = 1$ for all $\gamma \in \Gamma$, then $v(\psi) = 1$.

Fuzzy Logics and the Sorities

For $Ł_c$, modus ponens fails and this blocks the paradox.

For $Ł_\aleph$, take a Sorites series starting at 0 and ending at 100.
We know that $v(\phi(0)) = 1$ and $v(\phi(100)) = 0$.

We can model the series as $v(\phi(k)) = 1 - k/100$

Premises of the form $\phi(k) \to \phi(k+1)$ have values very close to 1 (in particular, $99/100$)

Assessing the Situation

The $Ł_c$ answer to the sorites is: the sorites is not valid (modus ponens fails)

The $Ł_\aleph$ answer to the sorites is: some of the premises are not 'fully' true.

Truth and Degrees

Is *Amsterdam is a beautiful city* truer than *New York is a big city* ?

If John is happy, then John is happy $p \to p$

If John is happy, then John is not happy $p \to \neg p$

If $v(p) = 0.5$, then two sentences will have the same truth degree. Is this plausible?

[Williamson 2002]

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First Degree Entailment (FDE)

Developed by Nuel Belnap in the 70’ to model reasoning in distributed systems.

4-valued logic: $1, b, n, 0$

1: *true*
b: ***b**oth true and false*
n: ***n**either true nor false*
0: *false*

How to deal with conflicting or incomplete information in a database?

FDE - Semantic Clauses

$\wedge$	1	b	n	0
1	1	b	n	0
b	b	b	0	0
n	n	0	n	0
0	0	0	0	0

$\vee$	1	b	n	0
1	1	1	1	1
b	1	b	1	b
n	1	1	n	n
0	1	b	n	0

$\neg$	
1	0
b	b
n	n
0	1

$p \rightarrow q \equiv \neg p \vee q$

Logical consequence is preservation of truth: $T^{+} = \{1, b\}$

FDE restricted to $\{1, n, 0\}$ gives us K_3^s.

FDE restricted to $\{1, b, 0\}$ gives us LP.

Outline

Making things precise

Supervaluationism was first introduced by van Fraassen (1966) and applied to the case of vagueness by Fine (1975).

The word *heap* is vague: there is no precise point at which a collection of grains becomes a heap.

However, there are ways to make this *precise*: for instance, we may take 'heap' as 'a collection of 1000 grains of sand'. We call this a *precisification* of heap.

For a word like *heap*, different precisification are admitted (e.g., 1000, 1001, . . .).

Semantic Indecision

The reason it's vague where the outback begins is not that there's this thing, the outback, with imprecise borders; rather there are many things, with different borders, and nobody has been fool enough to try to enforce a choice of one of them as the official referent of the word "outback." Vagueness is semantic indecision.

(Lewis 1986: *On the Plurality of Worlds*, p. 213)

[MacFarlane 2016]

Precisification

Let $v : P \to \{1, i, 0\}$ be a three-valued valuation. We say that a classical valuation v' is a *precisification* of v, and we write $v \leq v'$ iff

if $v(p) = 1$, then $v'(p) = 1$;
if $v(p) = 0$, then $v'(p) = 0$;
if $v(p) = i$, then $v'(p) = 1$ or $v'(p) = 0$.

	p	q
v	i	0

	p	q
v'_1	1	0
v'_2	0	0

Supertrue and Superfalse

A formula is *super-true* when it is true in all its precisification.

Given a three-valued valuation v, a formula ϕ is supertrue with respect to v iff $v'(\phi) = 1$ for all v' s.t. $v \leq v'$. We write $v \models^{!1} \phi$

A formula is *super-false* when it is false in all its precisification.

Given a three-valued valuation v, a formula ϕ is superfalse with respect to v iff $v'(\phi) = 0$ for all v' s.t. $v \leq v'$. We write $v \models^{!0} \phi$.

Logical Consequence

We can define both a *local* and a *global* notion of logical consequence, which are equivalent.

$\Gamma \models_g \phi$ iff for all three-valued valuations v if $v \models^{!1} \gamma$ for all $\gamma \in \Gamma$, then $v \models^{!1} \phi$.

$\Gamma \models_l \phi$ iff for all three-valued valuations v, for all v' s.t. $v \leq v'$, if $v'(\gamma) = 1$ for all $\gamma \in \Gamma$, then $v'(\phi) = 1$.

Supervaluationism and Team Semantics

In team semantics (Hodges 1997, Väänänen 2007) the satisfaction relation is given in terms of *a (non-empty) set V of classical valuations*:

$V \models p$	iff	$\forall v \in V : v(p) = 1$
$V \models \neg\phi$	iff	$\forall v \in V : \{v\} \not\models \phi$
$V \models \phi \wedge \psi$	iff	$V \models \phi$ and $V \models \psi$
$V \models \phi \vee \psi$	iff	$V' \models \phi$ and $V'' \models \psi$ with $V' \cup V'' = V$

$\Gamma \models \psi$ iff for any V s.t. for all $\gamma \in \Gamma$, $V \models \gamma$, then $V \models \psi$

We can make the connection with supervaluationism explicit, as for the language above it holds that:

$V \models \phi$ iff $\forall v \in V : \{v\} \models \phi$ (i.e., $v(\phi) = 1$)

$V \models p \vee q$ iff $\forall v \in V : v(p \vee q) = 1$

Bivalence and Law of Excluded Middle

Supervaluationism does not satisfy bivalence, in the sense that it is not the case that for any set of valuations V:

$V \models p$ or $V \models \neg p$

Or in terms of precisifications, that for any three-valued valuation v:

$v \models^+ p$ or $v \models^- p$

However, the law of excluded middle is valid.

$\models p \vee \neg p$

Moreover it holds that

$\Gamma \models_S \phi$ iff $\Gamma \models_{CL} \phi$

The Sorites

$$\begin{array}{l} \phi(1) \\ \phi(1) \to \phi(2) \\ \vdots \\ \phi(1M-1) \to \phi(1M) \\ \hline \phi(1M) \end{array}$$

Under a supervaluationist account, the conjunction of the conditional premises of the form $\phi(k) \to \phi(k+1)$ is superfalse.

But its negation $(\phi(1) \wedge \neg\phi(2)) \vee (\phi(2) \wedge \neg\phi(3)) \vee \ldots$ is supertrue.

Why is this not a problem?

Not all conditionals $\phi(k) \to \phi(k+1)$ are supertrue, but no such conditional is superfalse.

Assessing the Situation

The conditional premises of the Sorites are together superfalse.

The negation of the conditional premises of the Sorites is supertrue.

By looking at the first-order case, this means that $\forall n(\phi(n) \rightarrow \phi(n+1))$ is superfalse. Its negation $\exists n(\phi(n) \wedge \neg\phi(n+1))$ is supertrue.

But there is no d s.t. $\phi(d) \wedge \neg\phi(d+1)$ [for the same reason that $p \vee q$ holds, but p, q might not.]