Truthmakers

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Readings

Required:

- Lecture notes: chapter 5

Optional:

- Fine, K. (2017). *Truthmaker semantics*. A Companion to the Philosophy of Language, 556-577.

Plan

1. Truthmaking and satisfaction
2. Exact and Inexact Truthmaking
3. Lewis argument

Outline

1. Truthmaking and satisfaction
2. Exact and Inexact Truthmaking
3. Lewis argument

Truthmaking and satisfaction

A model is a set of facts $\mathcal{F}$, which comes with a part-whole structure $\sqsubseteq$ on $\mathcal{F}$ which is reflexive, antisymmetric and transitive (a partial order).

$\mathcal{F}$ is **closed under least upper bounds** with respect to $\sqcup$ (typically called *fusion*).

$\{\mathbf{p}\} \models_e p$ and $\{\overline{\mathbf{p}}\} =\!|_e p$ if p is atomic.

$$\begin{aligned} f \models_e \neg\phi \quad &\text{iff} \quad f =\!|_e \phi \\ f =\!|_e \neg\phi \quad &\text{iff} \quad f \models_e \phi \end{aligned}$$

$$\begin{aligned} f \models_e \phi \wedge \psi \quad &\text{iff} \quad \exists g, h : f = g \sqcup h,\ g \models_e \phi,\ h \models_e \psi \\ f =\!|_e \phi \wedge \psi \quad &\text{iff} \quad f =\!|_e \phi \text{ or } f =\!|_e \psi \end{aligned}$$

$$\begin{aligned} f \models_e \phi \vee \psi \quad &\text{iff} \quad f \models_e \phi \text{ or } f \models_e \psi \\ f =\!|_e \phi \vee \psi \quad &\text{iff} \quad \exists g, h : f = g \sqcup h,\ g =\!|_e \phi,\ h =\!|_e \psi \end{aligned}$$

One can prove by induction that $f \in T(\phi)$ iff $f \models_e \phi$.

Truthmaking and satisfaction (2)

This is the version used in the lecture notes (to ease your work, we adopt this for the exercises and the exam!)

A model is a triple $M = \langle S, \leq, I \rangle$

- S is a set of states.
- $\leq$ is a partial order such that any two $s, s' \in S$ have a lub $s \sqcup s'$.
- $I = (I^+, I^-)$ is a pair of functions $S \times P \to \{0, 1\}$ satisfying $\sqcup$-closure:

  If $I^+(s, p) = 1$ and $I^+(s', p) = 1$, then $I^+(s \sqcup s', p) = 1$

  If $I^-(s, p) = 1$ and $I^-(s', p) = 1$, then $I^-(s \sqcup s', p) = 1$.

Semantic Clauses

We define positive and negative truthmaking

$s \models^+ p$ iff $I^+(s, p) = 1$
$s \models^- p$ iff $I^-(s, p) = 1$.

$s \models^+ \neg\phi$ iff $s \models^- \phi$
$s \models^- \neg\phi$ iff $s \models^+ \phi$

$s \models^+ \phi \wedge \psi$ iff there are $s', s'' \in S$ with $s' \sqcup s'' = s$ and $s' \models^+ \phi$ and $s'' \models^+ \psi$.
$s \models^- \phi \wedge \psi$ iff $s \models^- \phi$ or $s \models^- \psi$ or there are $s', s'' \in S$ with $s' \sqcup s'' = s$ and $s' \models^- \phi$ and $s'' \models^- \psi$.

$s \models^+ \phi \vee \psi$ iff $s \models^+ \phi$ or $s \models^+ \psi$ or there are $s', s'' \in S$ with $s' \sqcup s'' = s$ and $s' \models^+ \phi$ and $s'' \models^+ \psi$.
$s \models^- \phi \vee \psi$ iff there are $s', s'' \in S$ with $s' \sqcup s'' = s$ and $s' \models^- \phi$ and $s'' \models^- \psi$.

Logical Consequence

$\Gamma \models \phi$ iff all models M and states s, if $s \models^+ \gamma$ for all $\gamma \in \Gamma$, then $s \models^+ \phi$.

An important difference

Compare the clauses of conjunction

$f \models_e \phi \wedge \psi$ iff $\exists g, h : f = g \sqcup h,\ g \models_e \phi,\ h \models_e \psi$

$f \dashv_e \phi \wedge \psi$ iff $f \dashv_e \phi$ or $f \dashv_e \psi$

$s \models^+ \phi \wedge \psi$ iff there are $s', s'' \in S$ with $s' \sqcup s'' = s$ and $s' \models^+ \phi$ and $s'' \models^+ \psi$.

$s \models^- \phi \wedge \psi$ iff $s \models^- \phi$ or $s \models^- \psi$ or there are $s', s'' \in S$ with $s' \sqcup s'' = s$ and $s' \models^- \phi$ and $s'' \models^- \psi$.

The former corresponds to an **exclusive** version, while the latter to an **inclusive** version.

A verifier for $\phi \vee \psi$ should also be a verifier for $\phi \wedge \psi$. A falsifier for $\phi \wedge \psi$ should also be a falsifier for $\phi \vee \psi$.

$f \dashv_e \phi \wedge \psi$ iff $f \dashv_e \phi$ or $f \dashv_e \psi$ or $f \dashv_e \phi \wedge \psi$

Examples

$p, q \models p$

$p \wedge q \not\models p$

$(p \vee q) \wedge (p \vee r) \not\models p \vee (q \wedge r)$

But $(p \wedge q) \vee (p \wedge r) \models p \wedge (q \vee r)$ [exercise]

Closure

One can show that formulas are closed under the fusion operator $\sqcup$.

For all formulas ϕ, all truthmaker models M, and states s and s':

- If $s \models^+ \phi$ and $s' \models^+ \phi,$ then $s \sqcup s' \models^+ \phi$.
- If $s \models^- \phi$ and $s' \models^- \phi,$ then $s \sqcup s' \models^- \phi$.

Exact vs Minimal Truthmaker

An exact truthmaker does not have to be minimal.

s minimally verifies ϕ if $s \models^+ \phi$ and for any $s' \sqsubseteq s$ s.t. $s' \models^+ \phi$, then $s' = s$.

In words, if s exactly verifies ϕ and no proper subpart of s exactly verifies ϕ.

Consider p 'It is cold' and $p \vee (p \wedge q)$ 'It is cold or (it is cold and it rains).'

With the previous notation, we have that $T(p) = \{\{p\}\}$, while $T(p \vee (p \wedge q) = \{\{p\}, \{p, q\}\}$

Thinking in terms of states, consider the state p and q as the sole verifiers of p and q. p would be the minimal verifier of both p and $p \vee (p \wedge q)$, even though the latter is also exactly verified by $\mathrm{p} \sqcup \mathrm{q}$.

Subject-Matter

Fine (2017) proposes an interesting notion of subject matter of a formula.

The subject matter of ϕ is the fusion $s_1 \sqcup s_2 \sqcup \ldots$ of its verifiers.

$$\sigma(\phi) = \bigvee(|\phi|^+)$$

Subject matters are states, rather than a relation between worlds (as in an intensional treatment).

$$\sigma(p \wedge q) = \sigma(p \vee q)$$

Outline

Exact Truthmaking

$\{\mathbf{p}\} \models_e p$ and $\{\overline{\mathbf{p}}\} \models_e p$ if p is atomic.

$$
\begin{aligned}
f \models_e \neg\phi \quad &\text{iff} \quad f \dashv_e \phi \\
f \dashv_e \neg\phi \quad &\text{iff} \quad f \models_e \phi \\[6pt]
f \models_e \phi \wedge \psi \quad &\text{iff} \quad \exists g, h : f = g \sqcup h,\ g \models_e \phi,\ h \models_e \psi \\
f \dashv_e \phi \wedge \psi \quad &\text{iff} \quad f \dashv_e \phi \text{ or } f \dashv_e \psi \\[6pt]
f \models_e \phi \vee \psi \quad &\text{iff} \quad f \models_e \phi \text{ or } f \models_e \psi \\
f \dashv_e \phi \vee \psi \quad &\text{iff} \quad \exists g, h : f = g \sqcup h,\ g \models_e \phi,\ h \models_e \psi
\end{aligned}
$$

Inexact Truthmaking

To capture tautological entailment, we need a notion of **inexact** truthmaking, $T^*(\phi)$.

$f \in T^*(\phi)$ iff $\exists g \subseteq f : g \in T(\phi)$ [Van Fraassen style]

$f \models_i \phi$ iff $\exists g \sqsubseteq f : g \models_e \phi$ [new style]

$\phi \models_i \psi$ iff (for all models): $T^*(\phi) \subseteq T^*(\psi)$
or equivalently
$\phi \models_i \psi$ iff (for all models): $\forall f,$ if $f \models_i \phi,$ then $f \models_i \psi$.

ϕ tautologically entails ψ iff $\phi \models_i \psi$

Inexact truthmaking and satisfaction

$f \models_i p$ iff $\mathbf{p} \in f$ $\quad f \dashv_i p$ iff $\overline{\mathbf{p}} \in f$, if p atomic.

$f \models_i \neg\phi$ iff $f \dashv_i \phi$

$f \dashv_i \neg\phi$ iff $f \models_i \phi$

$f \models_i \phi \wedge \psi$ iff $f \models_i \phi$ and $f \models_i \psi$

$f \dashv_i \phi \wedge \psi$ iff $f \dashv_i \phi$ or $f \dashv_i \psi$

$f \models_i \phi \vee \psi$ iff $f \models_i \phi$ or $f \models_i \psi$

$f \dashv_i \phi \vee \psi$ iff $f \dashv_i \phi$ and $f \dashv_i \psi$

$\Gamma \models \phi$ iff $(i) \forall f \in \mathcal{F},$ if $\forall \gamma \in \Gamma : f \models_i \gamma,$ then $f \models_i \phi,$ and
$(ii) \forall f \in \mathcal{F},$ if $f \not\models_i \phi,$ then $\exists \gamma \in \Gamma : f \not\models_i \gamma$

Defining worlds

A fact f is **maximal** iff for every $p \in SOA : p \in f$ or $\overline{p} \in f$

A fact f is **consistent** iff for no $p \in SOA : p \in f$ and $\overline{p} \in f$

A fact f is a **possible world** iff f is maximal and consistent

A fact f is an **impossible world** iff f is maximal but not consistent

A note on hyperintesionality

Think of worlds W as sets of maximally consistent facts: $\{p, \neg q, r, s, \neg t, \dots\}$

$[\![\phi]\!] =_{\text{df}} \{w \in W : \exists f \in T(\phi) : f \subseteq w\}$

Notice: although $[\![p \vee (p \wedge q)]\!] = [\![p]\!]$,
still $T(p \vee (p \wedge q)) = \{\{p, q\}\} \neq \{\{p\}\} = T(p)$

$\Rightarrow \quad T(\phi)$ is more fine-grained than $[\![\phi]\!]$

$T^*(\phi) = \{g \in F \mid \exists f \in T(\phi) : f \subseteq g\}$

Notice: although $[\![p \vee \neg p]\!] = [\![q \vee \neg q]\!]$,
still $T^*(p \vee \neg p) \neq T^*(q \vee \neg q)$

$\Rightarrow \quad$ even $T^*(\phi)$ is more fine-grained than $[\![\phi]\!]$

A note on negation

Do we have false makers? So far, we assumed that each state of affairs has exactly one negative counterpart.

We can relax this condition by assuming a primitive two-place relation $\perp$ between states of affairs.

What constraints on $\perp$?

- Symmetry: $a \perp b \Rightarrow b \perp a$

- Irreflexivity: $a \not\perp a$

- More?

Based on $\perp$ between states of affairs, we can derive $\perp$ between facts

$f \perp g$ iff $\exists a \in f, b \in g : a \perp b$

A note on negation

What about negative propositions?

$\overline{P} = \{f \in \mathcal{F} : \forall g \in P : f \perp g\}$

(An alternative route is to define a notion of negative fact, and negative proposition based on the latter).

Outline

Lewis Argument

1. $p \wedge \neg p$
2. p ($\wedge$ elimination)
3. $\neg p$ ($\wedge$ elimination)
4. $p \vee q$ ($\vee$ introduction)
5. q (disjunctive syllogism)

What to give up?

- $\wedge$ elimination
- $\vee$ introduction
- disjunctive syllogism
- transitivity of entailment

Disjunctive Syllogism

We have seen that **tautological entailment** rejects Disjunctive Syllogism.

Van Fraassen result: provide a truthmaker semantics for tautological entailment.

$\phi \models^T \psi$ iff $T^*(\phi) \subseteq T^*(\psi)$ iff $\forall f \in T(\phi) : \exists g \in T(\psi) : g \subseteq f$

$\vee$ introduction

Moving from p to $p \vee q$ adds an additional constant not present in the premise.

Parry (1932) formalized a system where ϕ **analytically entails** ψ only if all propositional variables in ψ are contained in ϕ

$\phi \models_a \psi$ iff
(i) ϕ classicaly entails ψ and
(ii) $\forall g \in T(\psi) : \exists f \in T(\phi) : g \subseteq f$

$\vee$ introduction

Possible problems:

- We lose some important meta-inferences

$\phi \models_a \psi \not\Rightarrow \neg\psi \models_a \neg\phi$

- It is a quite intuitive rule.

- We would need to reject sentences as the one below.

All husbands are spouses.

$\wedge$ elimination

This is a quite unnatural move.

But it can be implemented easily:

$\phi \models \psi$ iff $T(\phi) \subseteq T(\psi)$

Transitivity of entailment

1. $p \wedge \neg p$
2. p (∧ elimination)
3. $\neg p$ (∧ elimination)
4. $p \vee q$ (∨ introduction)
5. q (disjunctive syllogism)

(1) entails (2) entails (4)
(1) entails (3)
(3) and (4) entail (5)

Can we have that (1) does not entail (5)?

Transitivity of entailment

First attempt: we exclude all arguments with contradictory premises or tautologous conclusions.

However, we would like to have that $p \wedge \neg p$ entails $\neg p$.

We can accept the latter because it is a substitution instance of a valid argument without a contradictory premise (change $\neg p$ to q)

Smiley (1959), Tennant (1994): An argument is valid iff it is a substitution instance of an argument that

1. is classically valid
2. does not have a contradictory premise
3. does not have a tautologous conclusion.

Transitivity of entailment

However, do we want to accept $(p \wedge \neg p) \vee r \models q \vee r$?

This analysis can be (partly) captured using truthmakers as follows:

$$\phi \models^t \psi \iff \forall f \in T^{@}(\phi),\ \exists g \in T(\psi) \text{ such that } g \subseteq f,$$

where

$$T^{@}(\phi) = \begin{cases} \{f \in T(\phi) : \text{cons}(f)\}, & \text{if } \exists f \in T(\phi) \text{ such that cons}(f), \\ T(\phi), & \text{otherwise.} \end{cases}$$